Nondiffracting beam emission from hyperbolic metasurfaces

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Abstract
We demonstrate highly directive and transversely confined emissions from hyperbolic metasurfaces (HMSs) to free-space. The generation of such beams is attributed to the narrow hyperbolic isofrequency contour, such that only propagating waves with their wavevectors near normal incidence can transmit through the medium. The directions of transmitted waves’ Poynting vectors are altered by the medium and become parallel to the propagation direction, resulting in the generation of nondiffracting beams. We also suggest a realization of the proposed HMS using a layered silver-dielectric structure for optical wavelengths.

Keywords: metamaterial, hyperbolic metasurface, nondiffracting beam

(Some figures may appear in colour only in the online journal)

1. Introduction
Advances in metamaterials (MMs) over the past decade have brought unprecedented opportunities to the design of novel electromagnetic structures and devices. MMs are artificial periodic structures which possess extraordinary electromagnetic properties that cannot be found in naturally occurring materials [1]. Notable applications of MMs include superresolution imaging [2, 3], cloaking [4], and perfect wave absorption [5] etc. More recently, a subcategory of MM, the so-called hyperbolic metamaterials (HMMs) whose isofrequency surface is a hyperboloid, have gained considerable attention as they support waves with unbounded wavevectors. There have been works on demonstrating subwavelength imaging [6, 7], negative refraction [8], spontaneous emission enhancement [9], and lifetime engineering in HMMs [10]. Their widespread interest is due to the relative ease of nanofabrication, broadband operations, and frequency tunability. Hyperbolic metasurfaces (HMSs), similar to their bulk counterpart, have also been shown to possess exceptional abilities to control the flow of light, achieve anomalously large photonic density of states, and superresolution imaging [11]. These ultrathin low-loss structures allow cost-effective fabrications, and may hold promises for future applications in imaging, sensing, and quantum information processing etc.

Since the introduction of the Bessel beam which has a transverse intensity distribution independent on the propagation distance [12], the generation of nondiffracting (or diffraction-free) beams has been an active research area [13–15]. In this work, we apply a straightforward semi-analytical approach to analyze wave propagation through HMSs, and report on the generation of nondiffracting beams from HMS when its thickness is much smaller than the incident wavelength under low-loss conditions. For thicker structures, the generation of such transversely confined beams requires higher material loss to flatten the isofrequency surface, allowing wavevectors with larger oblique incident angles to pass through. As for the realization of the HMS, we suggest a layered silver-dielectric structure for optical wavelengths. It is worth mentioning that the generation of nondiffracting narrow beams is fundamentally different from the generation of spatially unconfining plane waves by isotropic epsilon-near-zero [16, 17] or zero index materials [18] studied previously. As will be shown in this paper, the reported emission effect offers greater simplicity and flexibility over the existing method of nondiffracting beam generation.
2. Nondiffracting beam emission from HMS

Here we focus on the transversely positive HMS [11] with its permittivity tensor and permeability specified as

\[
\begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}, \quad \mu = \mu_0.
\] (1)

where \( \varepsilon_x = \varepsilon_z > 0 \) and \( \varepsilon_y < 0 \). In contrast to transversely negative HMSs which support only high-\( k \) waves, the HMS considered here supports both high-\( k \) and low-\( k \) waves, and the latter is essential for generating nondiffracting beams, as will be shown later in this paper. The HMS is infinite along \( x \)- and \( z \)-directions, and has a finite thickness in \( y \)-direction. Due to the negative value of \( \varepsilon_y \), the HMS interacts strongly with \( p \)-polarized waves with three nonzero components in the above expression is restricted to shorten computation time, but a sufficient number are used to analyse the property of HMS. Using equation (2), the magnetic field at any location \((x, y)\) can be approximated as

\[
H_z(x, y) = H_0 \sum_{k_z} a_{k_z} e^{-jk_z(x-x_i)},
\] (7)

where \( a_{k_z} \) is the amplitude of each Fourier component, and \((x_i, y)\) is the location of the line source. The number of components in the above expression is restricted to shorten computation time, but a sufficient number are used to analyse the property of HMS. Using equation (2), the magnetic field can be expressed as

\[
H_z = H_0 e^{-jk_zx} \begin{cases} 
\varepsilon^{-jk_y} + R e^{jk_y}, & y < 0, \\
A e^{-jk_y} + B e^{jk_y}, & 0 \leq y \leq L, \\
T e^{-jk_y(y-L)}, & y > L,
\end{cases}
\] (2)

where \( H_0 \) is the magnitude of the incident wave, \( k_x \) and \( k_y \) are wave numbers along \( x \)- and \( y \)-directions in free-space, respectively (\( k_x^2 + k_y^2 = k_0^2 \) where \( k_0 \) is the free-space wave number), \( k_z \) is the wave number along \( y \)-direction inside the HMS, \( R \) and \( T \) are reflection and transmission coefficients, and \( A \) and \( B \) are the amplitudes of waves inside HMS traveling in forward and backward directions, respectively. Applying boundary conditions, the transmission and reflection coefficients are derived as

\[
T = \frac{2k_x k_y}{2k_x k_y \cos(k_yL) + j(k_x^2 + k_y^2) \sin(k_yL)},
\] (3)

\[
R = \frac{-j \sin(k_yL)(k_x^2 - k_y^2)}{2k_x k_y \cos(k_yL) + j(k_x^2 + k_y^2) \sin(k_yL)},
\] (4)

and \( A \) and \( B \) can be calculated from \( T \) and \( R \) as

\[
A = \frac{(1 + R)e^{jk_xL} - T}{2j \sin(k_yL)},
\]

\[
B = \frac{(1 + R)e^{-jk_xL} - T}{-2j \sin(k_yL)},
\] (5)

where \( k_y \) is given by the dispersion relation for the HMS (assuming \( k_z = 0 \)):

\[
\frac{k_x^2}{\varepsilon_x} + \frac{k_y^2}{\varepsilon_y} = k_0^2.
\] (6)

In order to analyze wave propagations through the HMS, we consider a discrete Fourier series to approximate the continuous spectrum of a localized line-source field

\[
H_y(x, y_i) = \sum_{k_z} a_{k_z} e^{-jk_z(x-x_i)},
\] (7)

The above expressions are numerical approximations of the magnetic field, as the source only contains a finite number of \( k_x \). Thus this approach is referred to as semi-analytical. Nonetheless, in this way we can reveal sufficient physical insight into the property of the HMS and analyze its interactions with incident waves.

We first consider an HMS with its thickness equal to \( \lambda/1000 \), where \( \lambda = 800 \) nm is the free-space wavelength. A single line source is excited at \( y = -100 \) nm. In the calculation, the components used in the expansions of (7) and (8) are \( k_x = -N \cdot dk_x, 0 \leq y \leq L, \) where \( N = 4000 \) and \( dk_x = 0.01 k_0 \). The isofrequency contours (IFCs) of the HMS with \( \varepsilon_x = 1, \ Re(\varepsilon_y) = -1 \times 10^{-5}, \) and varying imaginary part of \( \varepsilon_y \) is shown in figure 1(a). In the absence of material loss, the isofrequency surface is an extremely narrow hyperboloid, which only allows wavevectors in close proximity to the normal incidence to enter the HMS. Wave components with larger \( k_x \) encounter strong reflections due to the impedance mismatch, as shown by the transmission coefficient in figure 1(b). Thus the HMS in the \( \varepsilon_y \rightarrow 0^{-} \) regime acts as a spatial filter which only passes waves with small \( k_x \). Moreover, the directions of Poynting vectors transmitted inside the HMS are altered to become nearly parallel, due to the hyperbolic shape of the IFC. A remarkable result is that, when the thickness of the HMS is small, the Poynting vectors retain their directions after leaving the medium, resulting in a highly directive beam that has nearly no transverse variations in its intensity distribution. In other words, a nondiffracting beam is generated by the HMS, as illustrated by figure 1(c). When loss increases, the IFC becomes more and more flattened (see figure 1(a)), and the lack of mismatch for wave components with larger \( k_x \) allows them to enter the HMS. However, these waves cause the emitted beam from the HMS to be less spatially confined due to their nonparallel Poynting vectors after leaving the medium. Thus the generation of nondiffracting beams prefers low-loss and thin HMSs, as the transmission decreases with increasing slab thickness. For thick HMSs, it is still possible to generate transversely highly confined beams for a reduced propagation distance by increasing the material loss. Such effects are attributed to the
extraordinary transmission enhancement reported recently [19]. Figure 1(b) shows the increase in transmission coefficient when a larger amount of loss is introduced to the HMS. The emitted beam from a thick HMS (or HMM) with its thickness equal to $\lambda$ is shown in figure 1(d).

It is worth mentioning that in general there are three approaches to generating highly directive beams: (1) by blocking waves in other directions than the desired one; (2) by bending wavefronts, i.e. creating the focusing effect; and (3) by enhancing propagation in the desired direction (as obtained by exciting leaky waves [20–22]). For the generation of nondiffracting beams from HMS, its principle of operation mainly relies on the first approach. The relation to leaky wave excitation in the latter geometry of this paper will be further investigated.

To further analyze the interactions between incident waves and the HMS, we apply the semi-analytical method introduced above by specifying excitations with purely propagating or evanescent wave components. Figure 2(a) shows normalized magnetic field distribution for a $\lambda/1000$-thick lossless HMS when only exciting evanescent waves. It is shown that a majority of evanescent wave components are reflected by the HMS, and those that can pass through remain decaying in free-space. Moreover, no notable amplification of
evanescent waves is observed at the boundaries of the HMS. For the case of HMM (Figure 2(b)), although it is also highly reflecting, the Poynting vectors of the components entering the HMM are immediately directed to the \( y \)-direction. As a consequence, a transversely confined beam is generated. This allows us to conclude that the HMS operating in the proposed scheme mainly acts upon the propagating part of the incident wave, and thus such effect does not restrict the source to be in close vicinity of the HMS/HMM, as it is shown in Figure 2(b) that the source is located at one wavelength away from the structure, and a nondiffracting beam can still be generated. The above analysis demonstrates that both HMS and HMM can be used to generate transversely highly confined beams, however, as it may not be practical to simultaneously control the real and imaginary parts of the medium, for the realization of such devices, thin HMSs are preferred.

3. Numerical validation of nondiffracting beam emission from HMS

In order to validate the presented results, we perform full-wave numerical simulations using the finite element method (FEM) and investigate the effect of different parameters on the generated nondiffracting beams. The system configuration is the same as that in the previous analysis, except for the finite transverse dimension of the HMS along the \( x \)-direction. Figure 3 shows normalized magnetic field distributions for different transverse dimensions, \( w \) and thicknesses, \( L \) of the HMS. When the HMS is finite, corner diffractions from both ends along the transverse direction of the HMS turn to be an important factor. For small transverse dimensions, i.e. \( w < \lambda \), the corner diffractions become so prominent that the wavefront of the emitted wave remains to be nearly cylindrical, similar to the case for a single line source as shown in

**Figure 3.** Normalized magnetic field distributions from FEM simulations of HMSs with varying transverse dimension, \( w \) and thickness, \( L \). A magnetic line source is excited at a distance of \( \lambda/30 \) from the HMS. In all cases, \( \varepsilon_x = 1 \) and \( \text{Re}(\varepsilon_y) = -0.001 \). (a) \( w = \lambda \), \( L = \lambda/100 \), and \( \text{Im}(\varepsilon_x) = -0.0001 \). (b) \( w = 4\lambda \), \( L = \lambda/100 \), and \( \text{Im}(\varepsilon_x) = -0.0001 \). (c) \( w = 4\lambda \), \( L = \lambda/40 \), and \( \text{Im}(\varepsilon_x) = -0.001 \). (d) \( w = 4\lambda \), \( L = \lambda/40 \), and \( \text{Im}(\varepsilon_x) = -0.002 \), with a perfect electric conductor (PEC) placed at a distance of \( \lambda/10 \) from the HMS (on the same side as the source).
Thus one can expect that the emitted field approaches the infinite HMS case when the transverse dimension is increased. It is found that when \( \lambda > w \), the corner diffractions only provide negligible contributions to the emitted beam, and the transmitted wave changes to be highly directive and transversely confined, as shown in figure 3(b). Although the generation of nondiffracting beams prefers thin HMSs, the practical realization of the device depends on the current fabrication technology. Thus it is essential to investigate the effect of HMS’s thickness on the generated beams. Figure 3(c) shows that the transverse field confinement degrades slightly when the thickness of the HMS is increased from \( \lambda/100 \) to \( \lambda/40 \), and the loss is increased accordingly to maintain such a confinement, following our previous analysis. In practical applications, it is often desirable to generate nondiffracting beams only towards a single direction. This can be achieved by placing a perfect electric conductor (PEC) behind the source. The field distribution for such a configuration is shown in figure 3(d). By applying the PEC, the transverse field confinement is further enhanced. Nonetheless, due to corner diffractions, sidelopes appear at both sides of the main beam, which can be suppressed by increasing material loss (the thickness of HMS needs to be increased accordingly for generating nondiffracting beams), or placing impedance-matched absorbing materials to terminate the HMS in x-direction.

The above analysis only considers homogeneous HMS. For practical realizations, two types of structure are often considered: the layered metal-dielectric structure for transversely negative HMS, and metallic wires in a dielectric host for transversely positive HMS [11], with the former one being given more attention due to its relative ease of fabrication. In our case, the essential condition is to have negative and near-zero permittivity along the longitudinal direction. Thus the conventional layered metal-dielectric structure is rotated by 90° to form the structure as shown in figure 4(a). When appropriate material parameters are chosen, negative permittivity along both y- and z-directions can be achieved, and such a structure is suited for p-polarized waves. Applying the Maxwell Garnett theory, the effective permittivity can be calculated as [23]

\[
\epsilon_z = \frac{d_1 + d_2}{d_1 / \epsilon_1 + d_2 / \epsilon_2},
\]

\[
\epsilon_y = \epsilon_z = \frac{\epsilon_1 d_1 + \epsilon_2 d_2}{d_1 + d_2},
\]

where \( \epsilon_y \) can only be approximated by equation (9), as the thickness of the HMS is small compared to the wavelength. We assume that the proposed structure is formed by layered
silver embedded in air ($\varepsilon_2 = 1$). Thus at optical wavelengths

$$\varepsilon_1 = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 - j\gamma},$$

(10)

where $\varepsilon_{\infty} = 4.96$, $\omega_p = 1.36 \times 10^{16}$ rad s$^{-1}$ and $\gamma = 2.73 \times 10^{13}$ rad s$^{-1}$ [24] are plasma and collision frequencies, respectively. To achieve $\varepsilon_1 \approx 1$ and $\varepsilon_y \approx \varepsilon_z \to 0^-$, the thickness of each silver slab and the spacing between slabs are chosen to be $d_1 = 3$ nm and $d_2 = 83$ nm, respectively. The overall thickness of the structure in $y$-direction is $L = 60$ nm. The calculated relative permittivity according to equation (9) at the wavelength range of 700 nm to 900 nm is shown in figure 4(b). We can see that near 800 nm, the regime of $\varepsilon_y \to 0^-$ can be achieved. However, as we stated earlier, the permittivity along $y$-direction is only an estimate due to the finite size of the structure. Thus in simulations, the wavelength is varied to find the highest transverse confinement of the emitted wave. The structure modeled using FEM contains 31 layers of silver, with a total transverse dimension of 2580 nm. Perfect magnetic conductor conditions are applied to terminate the domain in $z$-direction. A magnetic line source is excited at a distance of 20 nm to the front interface of the structure. A PEC is placed at a distance of 70 nm behind the line source. The magnetic field intensity distribution at 624 nm is found to have the highest spatial confinement and plotted in figure 4(c). Such a significant blue-shift of the working wavelength is mainly due to the finite thickness of the structure. From our previous analysis, as the thickness of the structure in $y$-direction is about $\lambda/10$, a larger amount of loss is required to generate nondiffracting beams. However, figure 4(b) shows that the loss of the silver-air structure is extremely small. As a consequence, the propagation distance of the emitted beam is reduced. Nonetheless, the directivity is greatly enhanced compared with a PEC-backed line source, as shown in figure 4(d).

4. Conclusion

In conclusion, we have demonstrated nondiffracting beam emissions from HMS. Such effects are more pronounced when the thickness of the HMS is small. For thicker HMS, it is also possible to generate nondiffracting beams by increasing the material loss, which is attributed to the extraordinary loss-enhanced transmission effect. For the realization of the proposed HMS in optical wavelengths, we suggest a structure formed by layered silver slabs embedded in air. The proposed approach to generating nondiffracting beams using HMS offers greater simplicity and flexibility over the existing method of embedding a source inside a MM [25]. We anticipate that the reported emission effect may find its immediate applications in various fields at both microwave and optical frequencies, such as sensing and antenna design etc.

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